Probability II: B. Math (Hons.) I
Academic Year 2022-23, Second Semester
Midsem Exam
Total Marks = 50 Duration: 10:00 am - 12:30 pm

- **Show all your work and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.**
- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.**

1. Suppose (X, Y) is a continuous random vector with a joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} c & \text{if } 0 < x < 1,\, 1-x < y < 2-x, \\ 0 & \text{otherwise.} \end{cases}$$

   Here c is a positive constant.

   (a) (3 marks) Find c.

   (b) (12 marks) Compute marginal probability density functions of X and Y.

   (c) (5 marks) Calculate $P(X < Y)$.

2. Suppose X_1, X_2 are independent random variables such that $X_1 \sim Gamma(\alpha_1, \lambda)$ and $X_2 \sim Gamma(\alpha_2, \lambda)$. Define $Y_1 := X_1 + X_2$ and $Y_2 := \frac{X_1}{X_1+X_2}$.

   (a) (12 marks) Find a joint probability density function of Y_1 and Y_2.

   (b) (4 marks) Using (a), show that Y_1 and Y_2 are independent.

   (c) (4 marks) What are the marginal distributions of Y_1 and Y_2?

3. (10 marks) Let N be the number of empty poles when r distinguishable flags are displayed at random on n distinguishable poles (here $r, n \in \mathbb{N}$). Assuming that each pole has unlimited capacity, compute the expected value of N.